Klausur 16.3.2009

① a) $2^{567} = 10^{\lg(2)\cdot 567} = 10^{170.7} \Rightarrow \underline{\underline{171}}$ Stellen

$3^{1000} = 10^{\lg(3)\cdot 1000} = 10^{477.1} \Rightarrow \underline{\underline{478}}$ Stellen

b) $R = \frac{1}{1-e^{T/100}} \Leftrightarrow 1 - e^{T/100} = \frac{1}{R} \Leftrightarrow$

$-e^{T/100} = -1 + \frac{1}{R} = -\left(\frac{R-1}{R}\right) \Leftrightarrow$

$\frac{T}{100} = \ln\left(\frac{R-1}{R}\right)$

$R = 50: \quad T = 100 \ln\left(\frac{49}{50}\right) = \underline{\underline{-2.0203}}$

c) $(3\cdot 7 + 78^{925} - 4) \bmod 7 =$

$(0 + 1^{925} - 4) \bmod 7 = -3 \bmod 7 = \underline{\underline{4}}$

$((17\cdot 17)^{250} + 3 - 3\cdot 7) \bmod 4 =$

$((1\cdot 1)^{250} + 3 - 3\cdot(-1)) \bmod 4 = 7 \bmod 4 = \underline{\underline{3}}$

d) $\lim\limits_{x\to\infty}\left[\frac{(3x^2+4x)(x-1)-(3x^2+x)(x+1)}{(x+1)(x-1)}\right] =$

$\lim\limits_{x\to\infty}\left[\frac{\cancel{3x^3}+x^2-4x-(\cancel{3x^3}+4x^2+x)}{x^2-1}\right] =$

$\lim\limits_{x\to\infty}\left[\frac{-3x^2-5x}{x^2-1}\right] = \underline{\underline{-3}}$

$\boxed{\text{IK } 16.3.2009}$

② a) Spatprodukt = Volumen des durch Vektoren aufgespannten Spates (Parallelflachs)

b) nach 1. Spalte entwickeln

$$|\det A| = |(-1)[(-1)^{2k} z^2 - 1(1-z^2)]|$$
$$= |1 - 2z^2|$$

wird minimal (0) für $1 - 2z^2 = 0$

$$\Leftrightarrow \quad z = \pm \frac{1}{\sqrt{2}}$$

hat lokales Maximum 1 bei $z = 0$

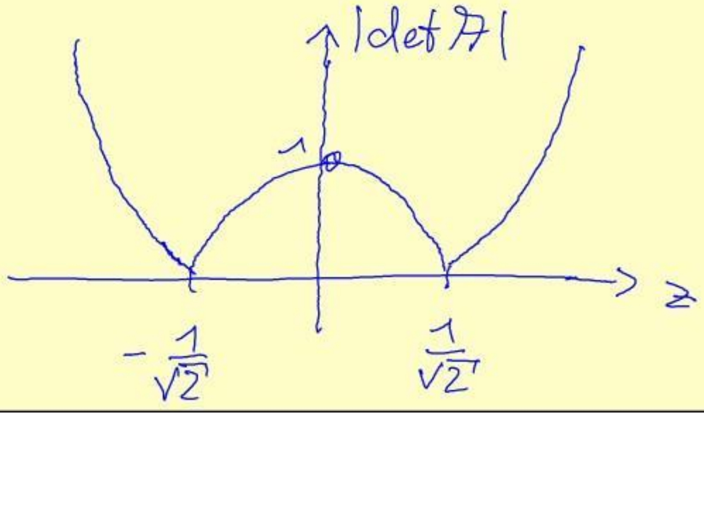
(denn für $f(z) = 1 - 2z^2$

ist $f'(z) = -4z = 0$ f. $z = 0$)

c) $\vec{a}_2 \cdot \vec{a}_3 = (-1)^k (1 - z^2) + (-1)^k z^2$

$= (-1)^k \cdot 1$ wird <u>nie</u> 0

$\Rightarrow \vec{a}_2$ <u>nie</u> senkrecht zu $\vec{a}_3$



$\boxed{\text{Klausur 16.3.2009}}$

(3) a) Fall 1: $x<0 \Rightarrow f(x) = -x$, $f'(x) = -1$

$$f''(x) = f'''(x) = 0$$

Fall 2: $x>0 \Rightarrow f(x) = \frac{x^2-2x}{x+2}$

$$f'(x) = \frac{(x+2)(2x-2) - 1\cdot(x^2-2x)}{(x+2)^2} = \frac{2x^2+2x-4-x^2+2x}{(x+2)^2}$$

$$= \frac{x^2+4x-4}{(x+2)^2}$$

$$f''(x) = \frac{(x+2)^{\cancel{2}}(2x+4) - 2\cancel{(x+2)}(x^2+4x-4)}{(x+2)^{\cancel{4}3}}$$

$$= \frac{\cancel{2x^2}+\cancel{8x}+8-\cancel{2x^2}-\cancel{8x}+8}{(x+2)^3} = \underline{\underline{\frac{16}{(x+2)^3}}}$$

$$f'''(x) = 16\cdot(-3)\cdot(x+2)^{-4} = \underline{\underline{\frac{-48}{(x+2)^4}}}$$

b) $\lim\limits_{x\to-\infty} f(x) = +\infty = \lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{x\to-2} f(x) = -2$

Fall 2: $f'(x) = 0 \Leftrightarrow x^2+2\cdot 2x+2^2-4-4=0$

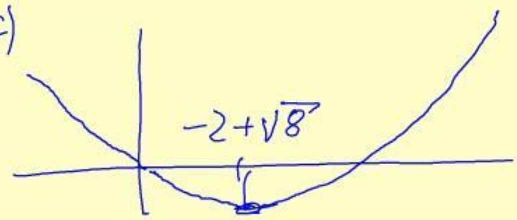
$$\Leftrightarrow (x+2)^2 = 8 \Leftrightarrow x_{1,2} = -2\pm\sqrt{8}$$

Nur $x_1 = -2+\sqrt{8}$ kommt in Frage

$f''(-2+\sqrt{8}) = \frac{16}{\sqrt{8}^3} > 0$, also $\underline{\underline{\text{Min. bei } x_1}}$

keine WP, da $f''(x)$ keine isolierten Nullstellen hat.

c)



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④ a) Normalenvektor

b) $$\left(\begin{array}{ccc|c} 1 & -2 & -4 & 1 \\ 1 & -4 & 1 & 1 \\ 1 & -6 & 6 & 1 \end{array}\right) \begin{array}{l} \\ \mid -Z_1 \\ \mid -Z_1 \end{array}$$

z.B. $\vec{n}_A \cdot \vec{x} = 1$

$\Leftrightarrow 1x_1 - 2x_2 - 4x_3 = 1$

$\rightarrow$ 1. Gleichung ✓

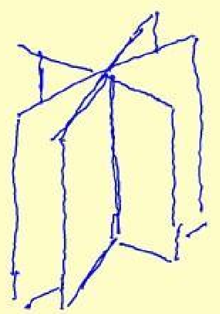
$$\left(\begin{array}{ccc|c} 1 & -2 & -4 & 1 \\ 0 & -2 & 5 & 0 \\ 0 & -4 & 10 & 0 \end{array}\right) \begin{array}{l} \\ \\ \mid -2Z_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -4 & 1 \\ 0 & -2 & +5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_3 \text{ bel.} \\ x_2 = \frac{5}{2} x_3 \\ x_1 - 2\left(+\frac{5}{2}\right)x_3 - 4x_3 = 1 \end{cases}$$

$\Leftrightarrow x_1 = 1 + 9x_3$

$$\boxed{L = \left\{ \vec{x} \,\middle|\, \vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} +9 \\ +5/2 \\ 1 \end{pmatrix} x_3 \,,\; x_3 \in \mathbb{R} \right\}}$$

c) Schnittmenge der drei Ebenen ist Gerade



d) Nein, sie sind entweder parallel oder identisch

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(5) Notwend. Bed. $W_d = 0 \quad \wedge \quad W_t = 0$

$W_d = 16d - d^2 = d(16-d) = 0 \iff d = 0 \vee d = 16$

$W_t = 6 - t = 0 \iff t = 6$

Hinreichende Bed. $\Delta > 0$, $W_{dd} < 0$ f. Max mit

$$\Delta = \begin{vmatrix} W_{dd} & W_{dt} \\ W_{td} & W_{tt} \end{vmatrix} = \begin{vmatrix} 16-2d & 0 \\ 0 & -1 \end{vmatrix} = 2d - 16$$

d	t	Δ	W_{dd}	
0	6	-16		$\Delta < 0$, kein Extremum
16	6	16	-16	$\Delta > 0 \Rightarrow$ Max

$W(16, 6) = 900.\overline{6}$

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(6) a) X ist (μ,σ)-normalverteilt $\Leftrightarrow$ $P(X \leq a)$ ist proportional einer Gaußglocke mit Mittelwert μ und Standardabweichung σ

b) $P(X \leq 5.5g) = 30\% \Rightarrow P\left(\frac{X-\mu}{\sigma} \leq \frac{5.5g - \mu}{1g}\right) = 30\%$

$\Rightarrow P\left(Z \leq 5.5 - \frac{\mu}{g}\right) = 30\% \Rightarrow \Phi\left(5.5 - \frac{\mu}{g}\right) = 30\%$

$\Rightarrow \Phi\left(\frac{\mu}{g} - 5.5\right) = 70\% \Rightarrow \frac{\mu}{g} - 5.5 = 0.52$

$\Rightarrow \mu = \underline{\underline{6.02\,g}}$

c) Null

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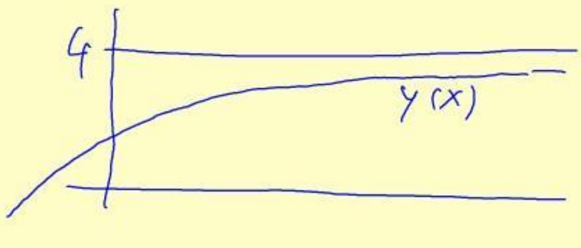
(8) a) 1. Ordn, implizit, linear, inhomogen (Begr. s. Schrift)

b) $y'(x) = \lambda a e^{\lambda x} \Rightarrow$

$y'(x) + 3y(x) = \lambda a e^{\lambda x} + 3a\,e^{\lambda x} \cancel{+12} = \cancel{12} \Rightarrow \underline{\underline{\lambda = -3}}$

$\boxed{y(x) = a e^{-3x} + 4}$

löst $y' + 3y = 12$



c) $y_s\left(\frac{1}{3}\right) = 4 + a e^{-1} = 3 \Rightarrow a = -1 \cdot e = -e$

$y_s(x) = -e\,e^{-3x} + 4 \Rightarrow \boxed{y_s(x) = 4 - e^{-3x+1}}$