a) $f(x,y,z) = 3x^2 - xy + 2yz + 4z^2$

$$dx = +0{,}25$$
$$dy = +0.25$$
$$dz = +0.25$$

$$f_x = 6x - y$$
$$f_y = -x + 2z$$
$$f_z = 2y + 8z$$

$$df = (6x-y)\cdot 0{,}25 + (-x+2z)\cdot 0{,}25 + (2y+8z)\,0{,}25$$
$$= (6\cdot 2 + 4)\cdot 0{,}25 + (-2-2)\cdot 0.25 + (2\cdot(-4) + 8\cdot(-1))\,0{,}25$$
$$= 4 - 1 - 4 = -1$$

b) $f(x_1, x_2, x_3, x_4) = x_1^4 + x_2^4 + x_3^4 + x_4^4$

$g(x_1, x_2, x_3, x_4) = x_1 + x_2 + x_3 + x_4 - 8 = 0$

$x = (x_1, x_2, x_3, x_4)$

Lagrange-Funktion:

$$L(x, \lambda) = f(x) + \lambda \cdot g$$

partielle Ableitungen

I $\quad L_{x_1} = 4x_1^3 + \lambda = 0$

II $\quad L_{x_2} = 4x_2^3 + \lambda = 0$

$\underline{III}$ $L_{x_3} = 4x_3^3 + \lambda = 0$

$\underline{IV}$ $L_{x_4} = 4x_4^3 + \lambda = 0$

$\underline{V}$ $L_\lambda = g = 0$

Aus $\underline{I} - \underline{IV}$ folgt: $x_1 = x_2 = x_3 = x_4$

in $\underline{V}$: $4x_1 - 8 = 0 \iff x_1 = 2$

damit Kandidat für einen Extremwert:

$(2, 2, 2, 2, 64)$