a) $$\int_0^{\pi/2} \sin^3 x \cdot \cos x \, dx$$

Substitution: $u = \sin x \qquad \frac{du}{dx} = \cos x$

$du = \cos x \, dx$

Substitution der Integrationsgrenzen:

$u(0) = \sin 0 = 0$

$u(\frac{\pi}{2}) = \sin \frac{\pi}{2} = 1$

$$\int_0^1 u^3 \, du = \left[\frac{u^4}{4}\right]_0^1 \underset{\uparrow}{=} \left[\frac{\sin^4 x}{4}\right]_0^{\frac{\pi}{2}}$$

Rücksubstitution

$$= \frac{\sin^4(\frac{\pi}{2})}{4} - \frac{\sin^4 0}{4} = \frac{1^4}{4} - 0 = \frac{1}{4}$$

b) $$V = \pi \cdot \int_1^2 (x^2+1)^2 \, dx$$

$$= \pi \int_1^2 (x^4 + 2x^2 + 1) \, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_1^2$$

$$= \pi \left(\frac{32}{5} + \frac{16}{3} + 2 - \frac{1}{5} - \frac{2}{3} - 1\right)$$

$$= \pi \left(\frac{31}{5} + \frac{14}{3} + 1 \right) = \pi \left(\frac{93 + 70 + 15}{15} \right)$$

$$= \frac{178}{15} \pi \approx 37.28 \text{ VE}$$

Beschreibung: Scheibe mit parabelförmiger Begrenzung!