Aufgabe 3

a)

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	e^x	1
1	e^x	1
2	e^x	1
3	e^x	1
4	e^x	1
5	e^x	1
6	e^x	1
7	e^x	1

$$T_6(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6$$

$$T_6(1) = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720}$$

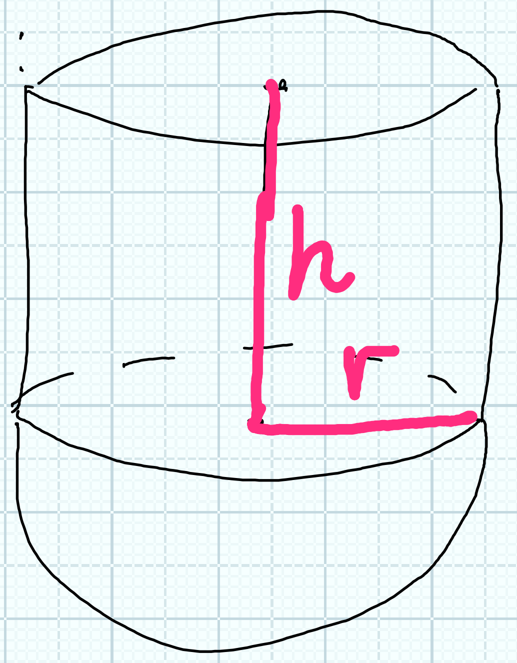
$$= 2{,}7180555$$

$$R_6(1) = \left| \frac{f^{(7)}(1)}{7!} \cdot 1^7 \right| < \frac{3}{7!} = 0.0005952$$

Genauigkeit auf 3 Dezimalstellen

Aufgabe 3b)

Skizze:



Oberfläche: $2\pi r^2 + 2\pi r \cdot h + \pi r^2 = 500\,cm^2$

Halbkugel Mantel Deckel

$$\Rightarrow \quad h = \frac{500 - 3\pi r^2}{2\pi r} = \frac{250}{\pi r} - \frac{3}{2} r$$

Nebenbedingung

Zielfunktion: $V(r,h) = \frac{2}{3}\pi r^3 + \pi r^2 h$

Halbkugel Zylinder

$$V(r,h) = \frac{2}{3}\pi r^3 + \pi r^2 \cdot \left(\frac{250}{\pi r} - \frac{3}{2} r\right)$$

$$= \frac{2}{3}\pi r^3 + 250 r - \frac{3}{2}\pi r^3$$

$$V(r) = -\frac{5}{6}\pi r^3 + 250 r$$

$$V'(r) = -\frac{15}{6}\pi r^2 + 250$$

$$V''(r) = -\frac{30}{6}\pi r = -5\pi r \qquad V''(r) < 0 \Rightarrow \text{MAX}$$

$$V'(r) = 0 \Leftrightarrow 250 = \frac{15}{6}\pi r^2 \Leftrightarrow r^2 = \frac{100}{\pi}$$

$$\underset{r>0}{\Rightarrow} \quad r = \sqrt{\frac{100}{\pi}} = \frac{10}{\sqrt{\pi}} \approx 5{,}641\,cm$$

In Formel für h einsetzen:

$$h = \frac{250}{\pi \cdot \frac{10}{\sqrt{\pi}}} - \frac{3}{2} \cdot \frac{10}{\sqrt{\pi}} \approx 5{,}641\,cm$$

$$\Rightarrow \quad h = r$$