

Aufgabe 8

$$a) z = 2\sqrt{2} + 2\sqrt{2}i \in \mathbb{C}$$

$$(1) |z| = \sqrt{2^2 \cdot 2 + 2^2 \cdot 2} = \sqrt{16} = 4$$

$$\varphi = \arctan \frac{2\sqrt{2}}{2\sqrt{2}} = \arctan 1 = \frac{\pi}{4} (=45^\circ)$$

$$z^7 = |z|^7 \cdot (\cos(7 \cdot 45^\circ) + i \sin(7 \cdot 45^\circ))$$

$$= 4^7 \cdot (\cos(315^\circ) + i \sin(315^\circ))$$

$$= 16384 (0.7071 + i(-0.7071))$$

$$= 11585,23 - 11585,23i$$

$$(2) |z| = \sqrt{8^2 \cdot 2 + 8^2 \cdot 2} = \sqrt{256} = 16$$

$$\varphi = \arctan \frac{8\sqrt{2}}{8\sqrt{2}} = \arctan 1 = \frac{\pi}{4} (=45^\circ)$$

4 Wurzeln: Betrag aller Wurzeln: $\sqrt[4]{16} = 2$

$$1. \quad 2 \cdot \left(\cos\left(\frac{45^\circ}{4}\right) + i \sin\left(\frac{45^\circ}{4}\right) \right)$$

$$= 2 \cdot (\cos(11.25^\circ) + i \sin(11.25^\circ))$$

$$= 1.96157 + 0.3901i$$

$$2. \quad 2 \cdot (\cos(11.25^\circ + 90^\circ) + i \sin(11.25^\circ + 90^\circ))$$

$$= -0.3901 + 1.96157i$$

$$3. \quad 2 \cdot (\cos(11.25^\circ + 180^\circ) + i \sin(11.25^\circ + 180^\circ))$$

$$= -1.96157 - 0.3901i$$

$$4. \quad 2 \cdot (\cos(11.25^\circ + 270^\circ) + i \sin(11.25^\circ + 270^\circ)) \\ = 0.3901 - 1.96157i$$

$$b) \quad |z_1| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$|z_2| = \sqrt{a^2 + 4}$$

$$\varphi_{z_1} = \arctan \frac{4}{3} = 53.13^\circ$$

$$\varphi_{z_2} = \arctan \frac{2}{a} = 53.13^\circ + 72^\circ = 125.13^\circ$$

$$\tan 125.13^\circ = -1.421 = \frac{2}{a}$$

$$\Rightarrow a = -1.407$$