V2024-05-06

Montag, 6. Mai 2024 10:56

Orga Klausureinsicht 8.5., 12^{45} – 13^{30}
R. 3230

HIP-Woche (13.5. –) MA2 V + Ü

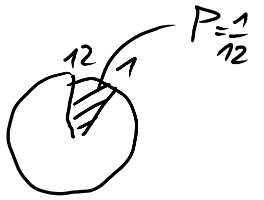
Mo 15 Uhr Tutorium

Wdh. Zuf. variable $\omega \longrightarrow (x_m, p_m)$

Erwartungswert ω: zufäll. Ereignis; x_m: Wert, p_m: Wahrsch

$$E(X) = \sum_m x_m p_m \qquad \underbrace{(x_m, p_m)}_{\text{Zufallsvar.}}$$

erwartete Wert, mittlerer Wert

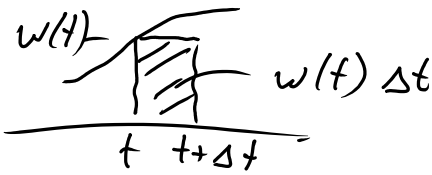


$w(t)$: die Fkt, die die infinitesimal schmalen Balken verbindet

Problem stetige Zuf.var. $P(X = t) = 0$ (!)

$$P(t < X \leq t + \Delta t) \approx w(t)\Delta t$$

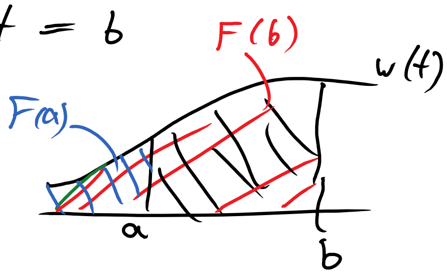
$$\xrightarrow{\Delta t = dt \to 0} P(t < X \leq t + dt) = w(t)dt$$



Für größere Bereiche $t = a$, $t + dt = b$

$$P(a \leq X \leq b) = \int_a^b w(t)\,dt$$

= Fläche unter Kurve $w(t)$



$$\left.\begin{array}{l} F(b) = P(X \leq b) \\ F(a) = P(X \leq a) \\ \boxed{a < b} \end{array}\right\} \Rightarrow F(b) - F(a) = P(X \leq b) - P(X \leq a) = P(a < X \leq b)$$

$$= \int_a^b w(t)\, dt$$

$\boxed{F(t) \text{ ist die Stammfunktion zu } w(t)}$

Übergang diskret -> stetig in Formeln

diskret: $F(b) = P(X \leq b) = \sum_{x_m \leq b} p_m$

$\downarrow \, dt \to 0$

stetig: $F(b) = P(X \leq b) = \int_{-\infty}^{b} \overbrace{w(t)\,dt}$

Erwartungswert

diskret: $E(X) = \sum_m x_m \, p_m$

$\downarrow \, dt \to 0$

stetig: $\boxed{E(X) = \int_{-\infty}^{\infty} t \, \overbrace{w(t)\,dt}}$

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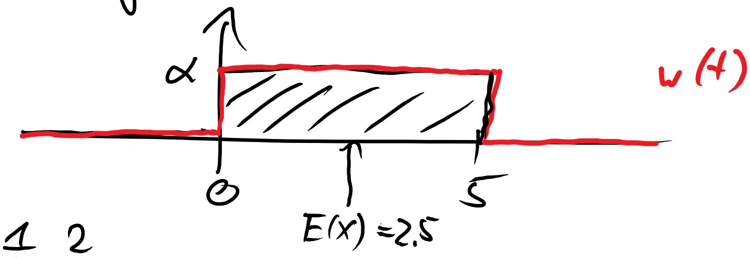
Beispiel stetige Zufallsvar.

X: in $[0, 5]$ gleichverteilt

a) Zeichne $w(t)$



Wieso gilt $\alpha = \frac{1}{5}$?

Weil $\int_{-\infty}^{\infty} w(t)\,dt = 1 \iff \int_0^5 w(t)\,dt = \alpha \cdot 5 = 1$

$\iff \alpha = \frac{1}{5}$

$$w(t) = \begin{cases} \frac{1}{5} & \text{f. } 0 \le t \le 5 \\ 0 & \text{sonst} \end{cases}$$

b) Welchen Erwartungswert hat X?

Vermutung 2.5

Nachrechnen $E(X) = \int_{-\infty}^{\infty} t\, w(t)\,dt = \frac{1}{5}\int_0^5 t\,dt$

$= \frac{1}{5}\,\frac{1}{2}t^2\Big|_0^5 = \frac{25}{10} = \underline{2.5} = \mu$

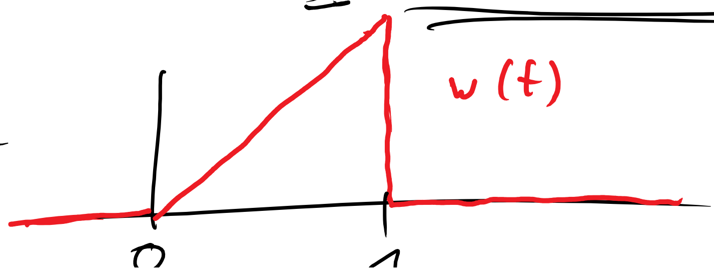
c) Welche Varianz hat X?

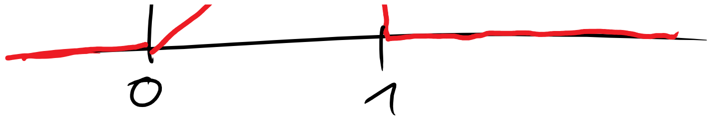
$$V(X) = E\left((X-\mu)^2\right) = \int_{-\infty}^{\infty} (t-\mu)^2\, w(t)\,dt$$

$$= \frac{1}{5}\int_0^5 (t-2.5)^2\,dt = \frac{1}{5}\,\frac{1}{3}(t-2.5)^3\Big|_0^5$$

$$= \frac{1}{15}\left[2.5^3 - (-2.5)^3\right] = \frac{2}{15}\cdot 2.5^3$$ = 25/12 = 2.0833

Übung $w(t) =$

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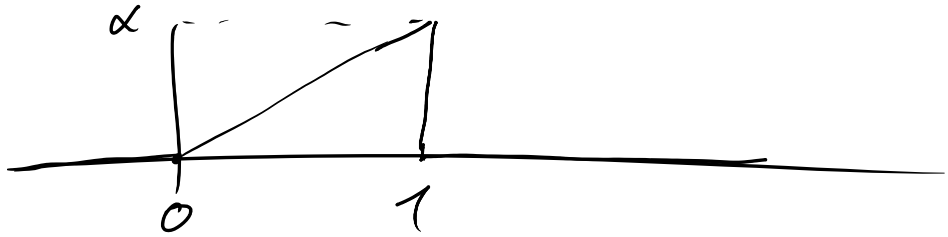
Übung

$$w(t) = \begin{cases} \alpha t & \text{f. } 0 \le t \le 1 \\ 0 & \text{sonst} \end{cases}$$



a) Was ist α :

$$\int_{-\infty}^{\infty} w(t)\,dt = 1$$

$$\Leftrightarrow \int_0^1 \alpha t\,dt = 1$$

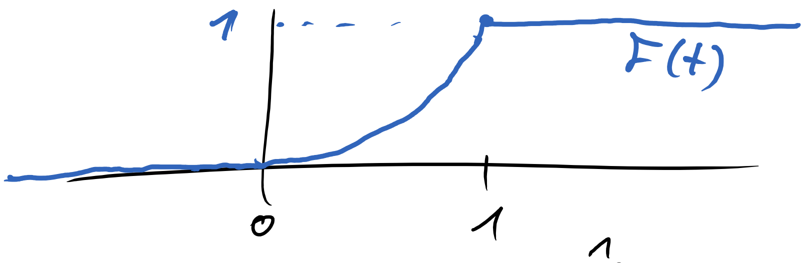
$$\Leftrightarrow \alpha \frac{1}{2} t^2 \Big|_0^1 = 1$$

$$\Leftrightarrow \boxed{\alpha = 2}$$

$$\boxed{w(t) = 2t \quad \text{für } 0 \le t \le 1}$$

$$F(t) = \int_{-\infty}^{t} w(t')\,dt' = \int_0^t 2t'\,dt' = t'^2 \Big|_0^t = t^2$$

für $0 \le t \le 1$



Was ist $E(X) = \int_0^1 t\, w(t)\,dt = \int_0^1 2t^2\,dt$

$$= \frac{1}{3} \cdot 2\, t^3 \Big|_0^1 = \frac{2}{3} = 0.\overline{6}$$