V 20.5.15 MA 2

- Tutorium → findet Do 21.5. statt $9^{00} - 10^{30}$, 3.104
- ab Mi 27.5 → Ane Schmitter V+Ü
- meinprof.de → bitte Bewertung abgeben
- Jobbörse: 3. Juni, 10 - 16:30

Komplexe Zahlen

$$re^{i\varphi} = r(\cos(\varphi) + i\sin(\varphi))$$ Eulersche Formel

$$e^{i\pi} = -1$$

Division in Polarkoordinaten

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\varphi_1}}{r_2 e^{i\varphi_2}} = \left(\frac{r_1}{r_2}\right) e^{i\varphi_1} e^{-i\varphi_2} = \left(\frac{r_1}{r_2}\right) e^{i(\varphi_1 - \varphi_2)}$$

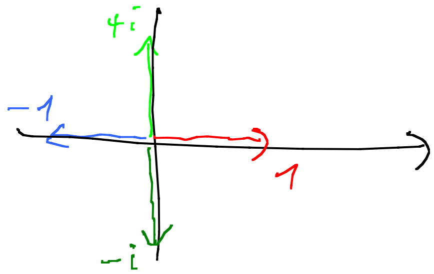
Quadrat $z^2 = z \cdot z$ Phase verdoppelt

$z^3 = z \cdot z \cdot z$ Phase verdreifacht

Potenzen und Wurzeln komplexer Zahlen

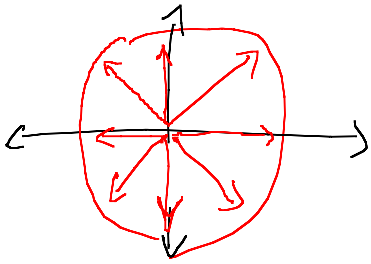
Im Komplexen: Wieviele Lösungen hat $z^4 = 1$?

$\Leftrightarrow \quad 1^{\frac{1}{4}} = z$



4. Wurzel hat 4 Lösungen

$\mathbb{L} = \{1, i, -1, -i\}$



$z^8 = 1 \Leftrightarrow z = 1^{\frac{1}{8}}$

8. Wurzel hat 8 Lösungen

Allgemein: bei n. Wurzel gibt es n Lösungen die den Vollkreis in n Sektoren teilen

Beispiel zu z^c, c rational

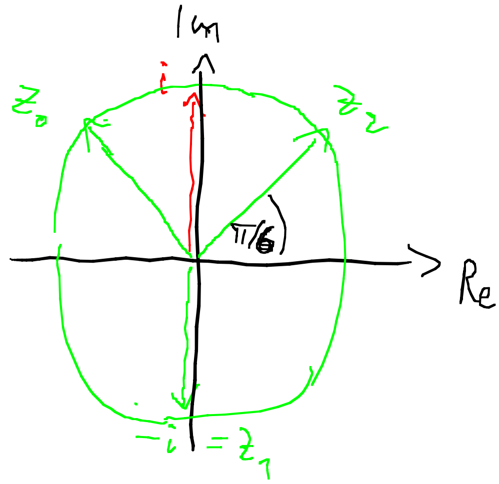
Was ist $i^{\frac{5}{3}}$? $\longrightarrow$ i in Exponentialform

$\Rightarrow i = e^{i\frac{\pi}{2}}$

$$z = i^{\frac{5}{3}} = \left(e^{i\frac{\pi}{2}}\right)^{\frac{5}{3}} = \left(\left(e^{i\frac{\pi}{2}}\right)^5\right)^{\frac{1}{3}}$$

$$= \left(e^{i\left(\frac{5}{2}\pi + 2k\pi\right)}\right)^{\frac{1}{3}} = \underline{\underline{e^{i\frac{5}{6}\pi + \frac{2k\pi}{3}}}}, \quad k = 0,1,2$$

$\Rightarrow$ drei Lösungen z_0, z_1, z_2



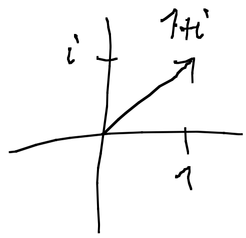
$$z_0 = e^{i\frac{5}{6}\pi} = -0.866 + 0.5i$$

$$z_1 = e^{i\frac{9}{6}\pi} = -i$$

$$z_2 = \underbrace{e^{i\frac{13}{6}\pi}}_{= e^{i\pi/6}} = 0.866 + 0.5i$$

$(\hat{=}\, 33^\circ)$

Übung Was ist $z = (\underbrace{1+i}_{u \rightarrow})^{3/4}$?



$u = 1+i$ in Exponentialdarstellung

$$r = \sqrt{Re(u)^2 + Im(u)^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\varphi = \arctan\left(\frac{Im(u)}{Re(u)}\right) = \arctan(1) = \frac{\pi}{4}$$

$$u = \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$z = u^{\frac{3}{4}} = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^{\frac{3}{4}} = \left(\left(2^{\frac{1}{2}} e^{i\frac{\pi}{4}}\right)^3\right)^{\frac{1}{4}} = \left(2^{\frac{3}{2}} e^{i\left(\frac{\pi}{4}\cdot 3 + 2k\pi\right)}\right)^{\frac{1}{4}}$$

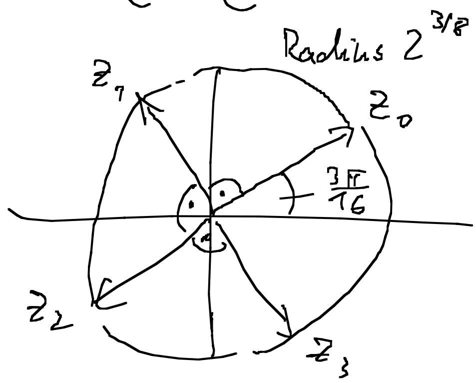
$4-1$

$$= 2^{\frac{3}{2}\cdot\frac{1}{4}}\, e^{i\left(\frac{3}{16}\pi + 2k\pi\cdot\frac{1}{4}\right)} = 2^{\frac{3}{8}}\, e^{i\left(\frac{3}{16}\pi + \frac{8}{16}\pi k\right)}, \quad k = 0,1,2,3$$

$$z_0 = 2^{\frac{3}{8}}\, e^{i\frac{3}{16}\pi}$$

$$z_1 = 2^{\frac{3}{8}}\, e^{i\frac{11}{16}\pi}$$

$\vdots$



$$\frac{3\pi}{16} \approx 35^\circ$$

Fundamentalsatz

Beispiel $z^3 - z^2 + 4z - 4 = (z-1)(z^2+4)$

$\overset{?}{=} (z-1)(z-2i)(z+2i) \rightarrow$ 3 Lösungen

2. Beispiel: $z^2 - (4-2i)z + (5-4i) = 0$

quadr. Ergänzung $\Leftrightarrow \underbrace{z^2 - 2(2-i)z + (2-i)^2}_{(z-(2-i))^2} - (2-i)^2 + (5-4i) = 0$

$\Leftrightarrow (z-(2-i))^2 - (4-1-2i-2i) + (5-4i) = 0$

$\Leftrightarrow (z-(2-i))^2 + 2 = 0$

$\Leftrightarrow (z-(2-i))^2 = -2 \quad | \sqrt{\ }$

$\Leftrightarrow z-(2-i) = \pm\sqrt{2}\, i$

$\Leftrightarrow \underline{\underline{z_{1,2} = (2-i) \pm \sqrt{2}\, i}}$