V MA 2 – 11.05.2016

$$\cos\varphi + i\sin\varphi = e^{i\varphi}$$

$$\cos z + i\sin z = e^{iz}$$

Umrechnungen

1) Polar → kartesisch

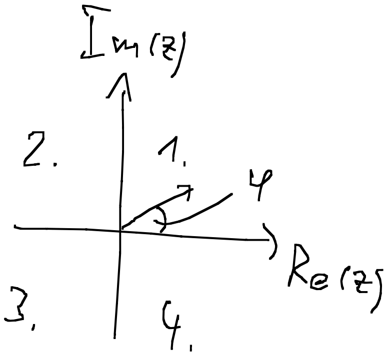
Geg r, φ → Gesucht x, y

einfach: $x = r\cos\varphi$, $y = r\sin\varphi$

2) kartesich → polar

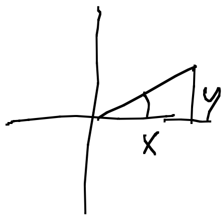
Geg. x, y → Gesucht r, φ

$$r = \sqrt{x^2 + y^2}$$



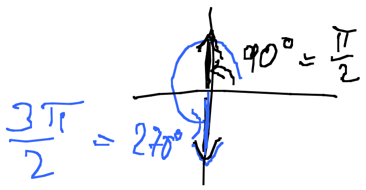
Achtung bei φ:

a) 1. od. 4. Quadrant ($Re(z) > 0$): $\varphi = \operatorname{atan}\left(\frac{y}{x}\right)$



b) 2. od. 3. Quadrant ($Re(z) < 0$): $\varphi = \operatorname{atan}\left(\frac{y}{x}\right) + \pi$

c) $\overbrace{Re(z)}^{x} = 0$: $\begin{cases} \varphi = \frac{\pi}{2} & \text{wenn } \underbrace{Im(z)}_{y} > 0 \\ \varphi = \frac{3\pi}{2} \text{ oder } -\frac{\pi}{2}, & \text{wenn } Im(z) < 0 \end{cases}$



In C, C++, Java, ... : $\operatorname{atan2}(y, x) \longrightarrow \varphi$

1) Beispiel: $z = \underbrace{-2}_{x} + \underbrace{2\sqrt{3}}_{y}\, i$ Umrechnung auf (r, φ):

$$r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$$

Welcher Quadrant? - 2. Quadrant $(x < 0)$

$$\varphi = \operatorname{atan}\left(\frac{2\sqrt{3}}{-2}\right) + \pi = -\frac{\pi}{3} + \pi = \frac{2}{3}\pi = 120°$$

2) Beispiel $z = 3e^{i\frac{5 \cdot 2\pi}{6}}$ → $\boxed{r = 3,\ \varphi = \frac{5 \cdot 2\pi}{6}}$

Umrechnung auf x, y

$$x = 3\cos\left(\frac{5 \cdot 2\pi}{6}\right) = 1.5$$

$$y = 3\sin\left(\frac{5 \cdot 2\pi}{6}\right) = -\frac{3}{2}\sqrt{3} = -2.598$$

3) Übung: Geg $x = 1.5$, $y \approx -2.598$ → Gesucht r, φ

Lösung: $r = \sqrt{x^2 + y^2} \approx 3$

φ: 4. Quadrant $(x > 0,\ y < 0)$

$$\varphi = \operatorname{atan}\left(\frac{y}{x}\right) = \operatorname{atan}\left(\frac{-2.598}{1.5}\right) = \underline{-1.047}$$

Ist das dasselbe wie $\varphi = \frac{5 \cdot 2\pi}{6} = 5.2359 \quad | -2\pi$

$$\varphi - 2\pi = \frac{5}{6}2\pi - 2\pi = -\frac{\pi}{6} = \underline{-1.047} \quad \boxed{\checkmark}$$

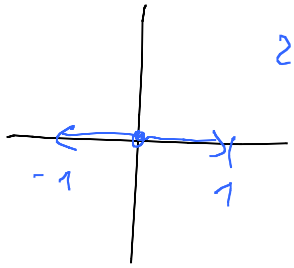
Potenzen komplexer Zahlen

Welche z erfüllen $z^3 = 1 = e^{i\cdot 0}$

(Welche Phasen φ ergeben bei $\varphi + \varphi + \varphi = 3\varphi$ eine 0 (oder 2π)?)

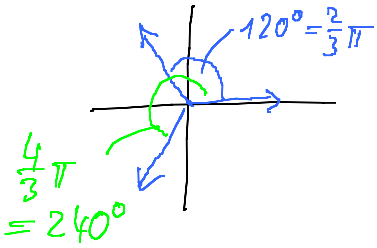
Welche z erfüllen $\to z^2 = 1$, $\to z^4 = 1$

$z^2 = 1$



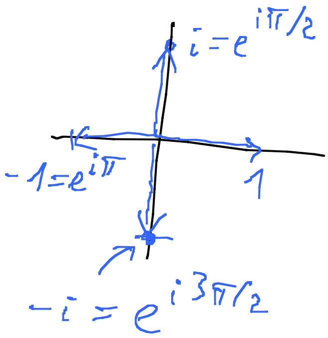
2 Lösungen
$\varphi = 0$
$\varphi = \pi$

$z^3 = 1$



3 Lösungen
$\varphi = 0$
$\varphi = \frac{2}{3}\pi$
$\varphi = \frac{4}{3}\pi$

$z^4 = 1$



$\varphi = 0$
$\varphi = \frac{\pi}{2}$
$\varphi = \pi$
$\varphi = \frac{3\pi}{2}$

Satz von Moivre

$$\boxed{\underbrace{(\cos\varphi + i\sin\varphi)^n}_{(e^{i\varphi})^n} = \underbrace{\cos(n\varphi) + i\sin(n\varphi)}_{e^{in\varphi}}}$$

$$(e^{i\varphi})^n = e^{in\varphi}$$

Übung: $(1+i)^{\frac{3}{4}}$ $\qquad 1+i = \sqrt{2}\, e^{i\frac{\pi}{4}}$

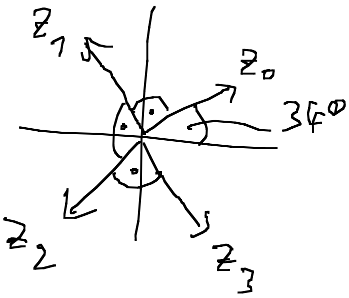
$$\left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^{\frac{3}{4}} = ?$$

$\frac{\pi}{4} = 45°$

$$= \left(\sqrt{2}^{3} e^{i\left(\frac{3\pi}{4} + 2k\pi\right)}\right)^{\frac{1}{4}}$$

$$= \sqrt{2}^{\frac{3}{4}} e^{i\left(\frac{3\pi}{16} + \frac{2k\pi}{4}\right)}$$

$$= \sqrt{2}^{\frac{3}{4}} e^{i\left(\frac{3\pi}{16} + \frac{k\pi}{2}\right)} \quad , k = 0,1,2,3$$



$\frac{3\pi}{4} = 135°$

$\frac{3\pi}{16} \approx 34°$