

Aufgabe 1

a)

A	B	$A \rightarrow B$	\overline{B}	$(A \rightarrow B) \wedge \overline{B}$	\overline{A}	$((A \rightarrow B) \wedge \overline{B}) \rightarrow \overline{A}$
0	0	1	1	1	1	1
0	1	1	0	0	1	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

A	B	C	$A \rightarrow B$	$B \rightarrow C$	$(A \rightarrow B) \wedge (B \rightarrow C)$	$A \rightarrow C$	$(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Stichwort : Tautologie

Aufgabe 1b)

$$\begin{aligned}
 & (12^5 \cdot 23^4) \bmod 5 \\
 &= (2^5 \cdot 3^4) \bmod 5 \\
 &= (32 \cdot 81) \bmod 5 \\
 &= (32 \bmod 5 \cdot 81 \bmod 5) \bmod 5 \\
 &= (2 \cdot 1) \bmod 5 \\
 &= 2
 \end{aligned}$$

Aufgabe 1c)

$$\begin{aligned}
 \lim_{n \rightarrow \infty} \left(\frac{1+6n+2n^2}{(n+3)n} \right) &= \lim_{n \rightarrow \infty} \frac{1+6n+2n^2}{n^2+3n} \\
 &= \lim_{n \rightarrow \infty} \frac{\frac{2n^2}{n^2} + \frac{6n}{n^2} + \frac{1}{n^2}}{\frac{n^2}{n^2} + \frac{3n}{n^2}} = \lim_{n \rightarrow \infty} \frac{2}{1} = 2
 \end{aligned}$$

(Note: In the original image, the terms $\frac{6n}{n^2}$, $\frac{1}{n^2}$, and $\frac{3n}{n^2}$ are circled in pink with arrows pointing to 0, indicating they vanish in the limit.)

Aufgabe 1d)

$$\lim_{x \rightarrow \infty} \left(x \cdot \ln \left(\frac{x+1}{x-1} \right) \right) = \lim_{x \rightarrow \infty} \frac{\ln \left(\frac{x+1}{x-1} \right)}{\frac{1}{x}}$$

führt auf einen unbestimmten Ausdruck,
Anwenden der Regel von de l'Hospital

$$= \lim_{x \rightarrow \infty} \frac{\left(\ln \left(\frac{x+1}{x-1} \right) \right)'}{(x^{-1})'} = \lim_{x \rightarrow \infty} \frac{\frac{1}{\frac{x+1}{x-1}} \cdot \frac{-2}{(x-1)^2}}{-\frac{1}{x^2}} \quad \left(\begin{array}{l} \text{innere} \\ \text{Abl. m.} \\ \text{Quotienten-} \\ \text{regel!} \end{array} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{x-1}{x+1} \cdot \frac{2}{(x-1)^2} \cdot x^2 = \lim_{x \rightarrow \infty} \frac{2x^2}{x^2-1} \stackrel{\text{erneut de l'Hosp.}}{=} \lim_{x \rightarrow \infty} \frac{4x}{2x}$$

$$= \lim_{x \rightarrow \infty} \frac{4}{2} = 2$$

(Note: The original image has "erneut de l'Hosp." written below the final limit.)